



Leonhard Euler 1707-1783

Switzerland's Foremost Scientific Expatriate

By Ronald S. Calinger & John Glaus

At the start of the 18th century, the Helvetic confederation into which Leonhard Euler was born was a breath of fresh air against the surrounding absolute monarchies. In five centuries of practical democracy, the Swiss cantons had attracted persons of skill and wealth and had become a redoubt for persecuted Protestants.

There is little information, anecdotal or otherwise, of Euler's early life however we know that he was born in Basel on April 15, 1707 (new style or Gregorian calendar) and should believe that he was raised a happy child. By the time he had reached 15 and was preparing to matriculate to



the University of Basel, he exhibited a precocious understanding of mathematics which his father allowed to develop with the great Johann Bernoulli I. Paul Euler was a Basler Reform pastor in Riehen.

About his mother Marguerite Bruckner, who was the daughter of a Basler minister, and two sisters little is known. In a tutorial, he impressed Johann Bernoulli who convinced Leonhard's father to end plans for his son to become a rural pastor and allow Leonhard, who was to become the most prolific mathematician in history, to transform the mathematical sciences through developing the exact sciences based on calculus.

At the age of nineteen, Euler received his Masters of Arts degree from the University of Basel. At the urging of Johann I Bernoulli, he applied for the open chair of physics there. As part of the application he wrote *De sono*, which became a classic and is often described as his doctoral dissertation. In April 1727 with no employment prospects in Basel, he left to join the Saint Petersburg Academy of Sciences. Johann I Bernoulli's sons, Daniel I and Nikolaus II had been hired by the Saint Petersburg Academy two years earlier. They had recommended Euler for the first open position. Since Empress Catherine I had died just before his arrival, the academy experienced financial difficulties. To assure an income, Euler served in the Russian Navy for three years. In 1730 he became professor of physics at the academy and in 1733, when Daniel Bernoulli returned to Basel, he gained the prestigious chair of mathematics. An improving financial situation allowed him to marry Katharina Gsell in 1734. They had fifteen children, of whom three boys and two girls survived him.

Euler began his phenomenal career in the mathematical sciences at the Imperial Academy of Sciences in St. Petersburg. During his first period from 1727 to 1741, his research, often in a preliminary stage, already ranged widely over arithmetic, astronomy, ballistics, cartography, differential geometry, elasticity, fluid and gas mechanics, hydraulics, infinite series, magnetism, music theory, number theory, optics, oscillations, rational mechanics and ship theory, along with state projects on fire engines, machines and science education.



He was to be both profound and prolific. The seventy-four volumes of the first three series of the Opera omnia all are 300-600 folio pages in length. These volumes demonstrate that the core of his research was in differential calculus and rational mechanics. In Saint Petersburg, Euler also chiefly gave close attention to music and ship theory. While in Saint Petersburg he was inventing two central branches of calculus; differential equations and the calculus of variations. His first milestone work, the two volumes *Mechanica* of 1736, was the first systematic effort to apply the new calculus to the physics in Isaac Newton's *Principia mathematica* and Jakob Hermann's *Phoronomia*. Euler invented many differential equations for this. The *Mechanica* largely brought him a distinguished European reputation.

After 1735, Euler suffered two health problems. That year he experienced extremely high fevers, possibly from scrofula, and in 1738 fevers and an abscess damaged the vision in his right eye, though apparently it did not cause total blindness in it. He blamed this situation on his study of maps and geography which was associated with Russia's Second Kamchatka expedition that gave Russia claim to Alaska. In his correspondence to Christian Goldbach, he said that the necessity to focus on minute details would prove to be harmful "der geographie is mir fatal."

The dynamics of push and pull brought Euler's first Saint Petersburg period to a close in 1741. The push was from Saint Petersburg itself, where xenophobic Russians erupted in violence against foreigners. The pull was from Frederick the

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}.$$

Series of Perfect Squares

Great of Prussia, who offered Euler an increased salary as well as safe haven in Berlin.

During his Berlin years from 1741-1766, Euler was at the peak of his career. During the early years of

his twenty-five year stay in Berlin, Euler wrote to Kaspar Wettstein a cousin and Baseler chaplain of the royal family in London that "...the king calls me his professor, and I think that I am the happiest man in the world."

Frederick had selected de Maupertuis to be the President of the newly reformed Academy of Science and Beaux Arts from the old Brandenburg Society of Sciences formed by Leibniz in 1700; Euler was selected as the director of the Mathematics class. While in Berlin, Euler wrote some 380 articles and completed several landmark books. His *Methodus inveniendi lineas curvas*,

$$\gamma = \lim_{n \rightarrow \infty} \left[\sum_{k=1}^n \frac{1}{k} - \ln(n+1) \right].$$

Gamma Constant

published in 1741, founded a semi geometric version of the calculus of variations and first demonstrated its extensive application to physics. Euler's two volumes *Introductio in analysin infinitorum* of 1748, a pre-calculus text, first based calculus on elementary functions rather than geometric curves. It, together with his two volume *Institutiones calculi differentialis* of 1755 and three volume *Institutiones calculi integralis*, published from 1768-1770, identified the central concepts of calculus, arranged them, added hundreds of his discoveries and provided a framework for the development of the field.

Euler was active in all aspects of the Berlin Academy. He supervised the observatory and the botanical gardens; selected the personnel; maintained the financial health of the academy in the sales of almanacs, the state lottery and the publication of maps. Frederick II also charged Euler with the solution to practical problems, such as correcting the level of the Finow canal, the work on the water pumps and pipes of the hydraulic system at Sans Souci; the Royal summer residence.



Together with his rivals Jean d'Alembert, Daniel Bernoulli, Alexis Clairaut and Colin Maclaurin, Euler founded modern mathematical physics and celestial mechanics based on calculus. Euler by now had been elected to the membership at the Royal Society of London and the Paris Academy of Sciences. He won the annual prize of the Paris Academy twelve times. As devout Christian, he defended traditional Protestant religions against the Wolffians freethinkers who rejected the concept of original sin. For recreation, he was a skilled chess player, enjoyed playing the clavier and was a chain pipe smoker.

$$\sum_{k=1}^{\infty} \frac{1}{k^2}$$

Summation of Infinite Series

There was more to Euler's achievement in Berlin. He developed a lottery to provide funds for war widows no doubt necessary because of Frederick's aggressive belligerence with all of Europe. His *Scientia navalis* of 1749 contributed to founding continuum mechanics, and his *Theoria motus corporum* of 1765 articulated the motion of rigid bodies. In the mid-1750's Euler withheld papers on the calculus of variations, giving his young colleague Louis de Lagrange full credit for inventing its modern form with new algorithms.

From 1753 to 1759 when de Maupertuis died, Euler had assumed responsibilities for the Academy without gaining the president's prestige or title. Euler's relationship to Frederick had soured by this time; but real bitterness came about when Frederick offered the presidency to Euler's mathematical nemesis Jean le Rond d'Alembert in 1763. D'Alembert politely but emphatically refused to leave Paris, but he also did so in deference to Euler's enormous reputation. Frederick continued to direct the Academy. But when the Russian troops entered Berlin in 1763 and pillaged Euler's Charlottenburg farm, the Empress instructed the Russian general to indemnify him completely. Euler had accepted a personal invitation from

Catherine II (the Great) and in 1766 he left Berlin to be the director of the Saint Petersburg Academy. Now, Euler was safely ensconced in a much safer Saint Petersburg with a very generous salary and positions for his sons.

During his second Saint Petersburg period from 1766 to 1783, Euler increased his productivity despite health setbacks. His *Vollständige Anleitung zur Algebra* appeared in 1770 with continuing fundamental contributions to number theory. His two volume *Letters to a Princess of Germany*, printed from 1768-1772 became the most successful high popularization of the sciences in the eighteenth century. His three volume *Dioptrica* appeared from 1769-1771. By this time Euler had lost sight in his right eye and a cataract operation which he failed to care for properly induced him to near total blindness. In May 1771 a disastrous fire struck Saint Petersburg and Euler barely escaped with his life. However, Peter Grimm, a family servant carried him to safety with most of his manuscripts. Euler simply saw his loss of sight as an inconvenience that would no longer bother him. His second lunar theory was dictated to Johann Albrecht and the 775 page *Theoria motum Lunae* was published in 1772. Three years after his wife Katharina died in 1773, a month before their fortieth wedding anniversary, Euler married her half sister Salome.

On September 18, 1783 Euler spent the first half of the day as usual. He gave a mathematics lesson to one of his grandchildren; then discussed with some friends the recently discovered planet

$$e^{i\pi} + 1 = 0.$$

Equation for Complex Variables

Uranus. About five o'clock in the afternoon he suffered a stroke and uttered only, "I am dying" before he lost consciousness. He died about eleven o'clock in the evening. Euler was buried in the Lutheran section of the Smolensk cemetery in Saint Petersburg. His literary legacy was so large that it took the Academy's journal fifty years to publish them.



Epilogue

Although he had been invited to return to Basel in 1748 to assume the Chair of Mathematics after Johann I Bernoulli's death, Euler's response was silence. Basel no longer provided an adequate stage for Euler, who worked in two capital cities of rising European powers. In addition, Euler may well have been upset that Basel had earlier refused to grant his wife citizenship or that he simply wished for Daniel Bernoulli to take his father's place.

If you go to Riehen, there is a plaque commemorating Euler's life on the Reformed Church, as elegant as it is simple:

„Leonhard Euler 1707-1783: Mathematiker, Physiker, Ingenieur, Astronome und Philosoph. Verbrachte in Riehen seine Jugendjahre. Er was ein grosser gelehrter und ein gütiger Mensch.“

Further Reading

- Ronald S. Calinger, Euler: The Presiding Mathematical Genius of the European Enlightenment, Princeton University Press 2007
- C. Edward Sandifer, The Early Mathematics of Leonhard Euler, Mathematical Association of America 2006
- Robert E. Bradley & C. Edward Sandifer, Editors,
• Studies in the History and Philosophy Mathematics, Elsevier Series Publishers 2006
- John S.D. Glaus, Euler and his Friends by G.-L. Dupasquier, Translated with
• Commentary, Jacques Hermann, Editeurs Paris 2007

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